Discrete Random Variables:
The Binomial and Geometric Distributions

A *dichotomous random variable* takes only the values 0 and 1. Let X be such a random variable, with $Pr(X=1) = p$ and $Pr(X=0) = 1-p$. Then $E[X] = p$, and $Var[X] = p(1-p)$.

Consider a sequence of n independent experiments, each of which has probability p of "being a success." Let $X_k = 1$ if the k-th experiment is a success, and 0 otherwise. Then the total number of successes in n trials is $X = X_1 + ... + X_n$; X is a *binomial random variable*, and

$$Pr(X = k) = \binom{n}{k} \cdot p^k (1-p)^{n-k} .$$

$E[X] = np$, and $Var[X] = np(1-p)$. These results follow from the properties of the expected value and variance of sums of (independent) random variables.

Next, consider a sequence of independent experiments, and let Y be the number of trials up to and including the first success. Y is a *geometric random variable*, and

$$Pr(Y = k) = (1-p)^{k-1} p .$$

$E[Y] = 1/p$, and $Var[Y] = (1-p)/p^2$.